Reconstruction of the Constitutive Parameters for an Ω Material in a Rectangular Waveguide

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***Abstract*—The inverse problem of determining the constitutive parameters of an Ω material is considered. The dispersive bi-anisotropic Ω sample is placed in a rectangular waveguide. All the constitutive parameters except one are reconstructed using the reflection and transmission data for some TE_{m0} and TE_{0n} modes. The remaining one can be obtained, e.g., from reflection of normally incident plane waves. Numerical results for the reconstruction are presented.**

I. Introduction

RECENTLY, considerable attention has been focused on wave propagation, scattering, and guidance in complex media in view of their potential usefulness in a variety of applications, e.g., control of absorption, shielding, and coating. Among these complex media, chiral media have been studied extensively in the past few years (e.g., [1]–[4]). A new class of complex materials, called Ω media, was introduced a few years ago [5], [6]. The properties of an Ω medium can be envisaged as arising from a distribution of metal half-loop with two extended arms. Some scattering properties and possible applications have been studied in [7]–[9] for an Ω material in which all the loops have their extended arms parallel to each other and in which all the normals to the planes of the loops are also parallel (cf. Fig. 1).

In the present paper we consider a homogeneous block of such an Ω material with their extended arms and normals to the planes of the loops pointing in the y and x directions, respectively, in a metallic rectangular waveguide along the z direction (see Fig. 1). The inner dimensions of the waveguide are $0 \leq x \leq a, 0 \leq y \leq b$, and the Ω sample occupies the region $0 \leq z \leq L$. We describe in the present paper how to reconstruct the constitutive parameters of the Ω material from measured reflection and transmission data for some modes at $z = 0$ and $z = L$. Determination of the constitutive parameters in waveguides has been studied for some dielectric media, see, e.g., [10], [11]. Considerable work has been done on determining the constitutive parameters of bi-isotropic chiral media using free-space measurements, see, e.g., [12], [13]. There are many advantages to the use of waveguide measurements in determining the parameters compared to free-space measurements, e.g., there is no problem of secondary diffraction effects. The constitutive relations for the Ω material are

$$\vec{D} = \bar{\bar{\varepsilon}}\vec{E} + \bar{\bar{\xi}}\vec{H} \tag{1}$$

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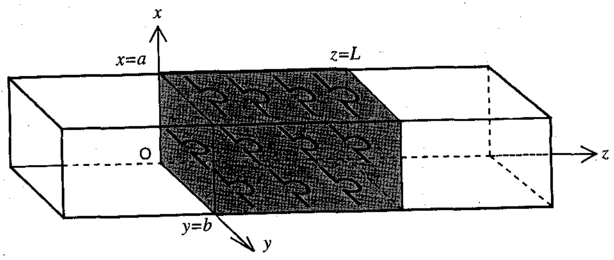

Fig. 1. An Ω sample in a rectangular waveguide.

$$\vec{B} = \bar{\bar{\mu}}\vec{H} + \bar{\bar{\zeta}}\vec{E} \tag{2}$$

where the parameter tensors have the following forms in the xyz coordinate system

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \qquad \bar{\bar{\mu}} = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix} \tag{3}$$

$$\bar{\bar{\xi}} = -j\sqrt{\varepsilon_0\mu_0} \begin{bmatrix} 0 & 0 & 0 \\ \Omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \bar{\bar{\zeta}} = j\sqrt{\varepsilon_0\mu_0} \begin{bmatrix} 0 & \Omega & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

and where ε_0 and μ_0 are the permittivity and permeability in vacuum, respectively, and Ω is the dimensionless parameter measuring the degree of cross-coupling between the electric and magnetic fields. Thus one sees that such an Ω material is bi-anisotropic. All the constitutive parameters for the Ω material are, in general, dispersive. The inverse problem is to determine the unknown dispersive parameters ε_i, μ_i ($i =$ 1, 2, 3), and Ω from reflection and transmission data of some modes.

II. Problem Formulation

Maxwell's equations in the xyz coordinate system with harmonic time dependence $\exp(j\omega t)$ are

$$\nabla \times \vec{E} = -j\omega(\bar{\bar{\mu}}\vec{H} + \bar{\bar{\zeta}}\vec{E}) \tag{5}$$

$$\nabla \times \vec{H} = j\omega(\bar{\bar{\varepsilon}}\vec{E} + \bar{\bar{\xi}}\vec{H}) \tag{6}$$

where the parameter tensors are given by (3) and (4), and $\vec{E} = (E_1, E_2, E_3)^T$, $\vec{H} = (H_1, H_2, H_3)^T$.

If the electromagnetic fields $\vec{E}$ and $\vec{H}$ for a given mode have a z-dependence of $\exp(-\gamma z)$, then from Maxwell's equations (5) and (6) one obtains the following equations:

$$\partial_x E_2 - \partial_y E_1 = -j\omega\mu_3 H_3 \tag{7}$$

$$\partial_x H_2 - \partial_y H_1 = j\omega\varepsilon_3 E_3 \tag{8}$$

$$\begin{bmatrix} -\gamma & 0 & 0 & j\omega\mu_2 \\ 0 & -\gamma+\omega\sqrt{\varepsilon_0\mu_0}\Omega & -j\omega\mu_1 & 0 \\ 0 & -j\omega\varepsilon_2 & -\gamma-\omega\sqrt{\varepsilon_0\mu_0}\Omega & 0 \\ j\omega\varepsilon_1 & 0 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} \partial_x E_3 \\ \partial_y E_3 \\ \partial_x H_3 \\ \partial_y H_3 \end{bmatrix}. \tag{9}$$

From (9) one can express the transversal fields in terms of the longitudinal fields as follows:

$$E_1 = \frac{-1}{\gamma^2+\omega^2\varepsilon_1\mu_2}[\gamma\partial_x E_3 + j\omega\mu_2\partial_y H_3] \tag{10}$$

$$E_2 = \frac{-1}{\gamma^2+\omega^2(\varepsilon_2\mu_1-\varepsilon_0\mu_0\Omega^2)} \times [(\gamma+\omega\sqrt{\varepsilon_0\mu_0}\Omega)\partial_y E_3 - j\omega\mu_1\partial_x H_3] \tag{11}$$

$$H_1 = \frac{-1}{\gamma^2+\omega^2(\varepsilon_2\mu_1-\varepsilon_0\mu_0\Omega^2)} \times [(\gamma-\omega\sqrt{\varepsilon_0\mu_0}\Omega)\partial_x H_3 - j\omega\varepsilon_2\partial_y E_3] \tag{12}$$

$$H_2 = \frac{-1}{\gamma^2+\omega^2\varepsilon_1\mu_2}[\gamma\partial_y H_3 + j\omega\varepsilon_1\partial_x E_3]. \tag{13}$$

Substituting (10)–(13) into (7) and (8), one obtains the following coupled equations for the longitudinal fields:

$$\left(\frac{\varepsilon_1}{h_{12}^2}\partial_x^2 + \frac{\varepsilon_2}{h_{21}^2}\partial_y^2 + \varepsilon_3\right)E_3 = \frac{1}{j\omega}\left(\frac{\gamma-\omega\sqrt{\varepsilon_0\mu_0}\Omega}{h_{21}^2} - \frac{\gamma}{h_{12}^2}\right)\partial_{xy}^2 H_3 \tag{14}$$

$$\left(\frac{\mu_1}{h_{21}^2}\partial_x^2 + \frac{\mu_2}{h_{12}^2}\partial_y^2 + \mu_3\right)H_3 = \frac{1}{j\omega}\left(\frac{\gamma+\omega\sqrt{\varepsilon_0\mu_0}\Omega}{h_{21}^2} - \frac{\gamma}{h_{12}^2}\right)\partial_{xy}^2 E_3 \tag{15}$$

where

$$h_{12}^2 = \gamma^2+\omega^2\varepsilon_1\mu_2$$
$$h_{21}^2 = \gamma^2+\omega^2(\varepsilon_2\mu_1-\varepsilon_0\mu_0\Omega^2). \tag{16}$$

The purpose of the present paper is to determine the parameters of the Ω material by measuring the reflection and transmission data at $z = 0$, L for certain modes. The analysis will become simpler if one uses some simple modes instead of hybrid modes. From (14) and (15) one sees that if $\Omega \neq 0$, decoupling of E_3 and H_3 occurs only when

$$\partial_x \equiv 0 \tag{17}$$

or

$$\partial_y \equiv 0. \tag{18}$$

Thus the decoupling occurs only for TE_{mn} modes with $m = 0$ or $n = 0$ (note that neither m nor n can be zero for TM modes in a rectangular waveguide, cf. [14]). In the next section, we calculate the reflection and transmission for these decoupled modes, which will be used in Section IV as the input to reconstruct the constitutive parameters of the Ω material.

III. Reflection and Transmission for TE_{m0} and TE_{0n} Modes

When $\partial_y \equiv 0$, one has the following decoupled equation and boundary condition for TE_{m0} modes (cf. (15) and (11))

$$\left(\partial_x^2 + \frac{\mu_3}{\mu_1}[\gamma^2+\omega^2(\varepsilon_2\mu_1-\varepsilon_0\mu_0\Omega^2)]\right)H_3 = 0 \tag{19}$$

$$E_2|_{x=0,a} = 0, \quad \text{i.e.,} \quad (\partial_x H_3)|_{x=0,a} = 0. \tag{20}$$

Thus one has the following set of solutions:

$$H_3^m = C_m \cos\left(\frac{m\pi x}{a}\right)\exp(-\gamma_m z)$$
$$E_2^m = -\frac{j\omega a\mu_3}{m\pi}C_m \sin\left(\frac{m\pi x}{a}\right)\exp(-\gamma_m z)$$
$$H_1^m = \frac{a\mu_3(\gamma m-\omega\sqrt{\varepsilon_0\mu_0}\Omega)}{m\pi\mu_1}C_m \sin\left(\frac{m\pi x}{a}\right)\exp(-\gamma_m z)$$
$$E_1^m = H_2^m = E_3^m = 0$$

where

$$\gamma_m = j\sqrt{\omega^2(\varepsilon_2\mu_1-\varepsilon_0\mu_0\Omega^2) - \frac{\mu_1}{\mu_3}\left(\frac{m\pi}{a}\right)^2} \tag{21}$$

and where we take the square root in (21) with nonnegative real part. Thus for frequencies larger than the cutoff frequency, the above set of solutions represent the ones that propagate in $+z$ direction (note that the time dependence is $\exp(j\omega t)$). Similarly, one can obtain another set of solutions to (19) and (20) which propagate in $-z$ direction. Putting these two sets of solution together, one can write the total tangential fields for TE_{m0} modes inside the waveguide as follows:

$$\vec{\boldsymbol{E}}_m^{\perp} = [E_m^+\exp(-\gamma_m z) + E_m^-\exp(\gamma_m z)] \times \sin\left(\frac{m\pi x}{a}\right)\boldsymbol{e}_y \tag{22}$$

$$\vec{\boldsymbol{H}}_m^{\perp} = \frac{1}{Z_m}[-E_m^+\exp(-\gamma_m z) + E_m^-\exp(\gamma_m z)] \times \sin\left(\frac{m\pi x}{a}\right)\boldsymbol{e}_x \tag{23}$$

($\boldsymbol{e}_x$, $\boldsymbol{e}_y$ are unit vectors in the x and y directions, respectively), where the constant amplitudes $E_m^{\pm}$ are to be determined by the boundary conditions at $z = 0$, L, and

$$Z_m = \frac{j\omega\mu_1}{\gamma_m - \omega\sqrt{\varepsilon_0\mu_0}\Omega}. \tag{24}$$

Assume that the waveguide is excited from the left region $z < 0$. Then the tangential fields for the TE_{m0}-modes, in the vacuum regions, have the forms

$$\left.\begin{aligned} \vec{\boldsymbol{E}}_m^{\perp} &= E_m^i[\exp(-\gamma_{0m}z) + R_m\exp(\gamma_{0m}z)]\sin\left(\tfrac{m\pi x}{a}\right)\boldsymbol{e}_y \\ \vec{\boldsymbol{H}}_m^{\perp} &= \tfrac{E_m^i}{Z_{0m}}[-\exp(-\gamma_{0m}z) + R_m\exp(\gamma_{0m}z)]\sin\left(\tfrac{m\pi x}{a}\right)\boldsymbol{e}_x \end{aligned}\right\}, \quad z < 0 \tag{25}$$

$$\left.\begin{aligned} \vec{\boldsymbol{E}}_m^{\perp} &= T_m E_m^i\exp[-\gamma_{0m}(z-L)]\sin\left(\tfrac{m\pi x}{a}\right)\boldsymbol{e}_y \\ \vec{\boldsymbol{H}}_m^{\perp} &= -\tfrac{T_m E_m^i}{Z_{0m}}\exp[-\gamma_{0m}(z-L)]\sin\left(\tfrac{m\pi x}{a}\right)\boldsymbol{e}_x \end{aligned}\right\}, \quad z > L \tag{26}$$

where

$$\gamma_{0m} = j\sqrt{\omega^2\varepsilon_0\mu_0 - \left(\frac{m\pi}{a}\right)^2} \qquad Z_{0m} = \frac{j\omega\mu_0}{\gamma_{0m}} \tag{27}$$

and R_m, T_m are the reflection and transmission coefficients, respectively, for the TE_{m0}-modes. E^i_m is the amplitude of the incident field. From (22), (23), (25), (26), and the continuity of the tangential fields at $z = 0$, L, one obtains

$$(1 + R_m)\cosh\gamma_m L - \frac{Z_m}{Z_{0m}}(1 - R_m)\sinh\gamma_m L = T_m \quad (28)$$

$$(1 + R_m)\sinh\gamma_m L - \frac{Z_m}{Z_{0m}}(1 - R_m)\cosh\gamma_m L = -\frac{Z_m}{Z_{0m}}T_m. \quad (29)$$

From the above two equations, one can uniquely determine the reflection coefficient R_m and the transmission coefficient T_m in a direct problem if the material parameters are known. One obtains

$$R_m = \frac{\left[\left(\frac{Z_m}{Z_{0m}}\right)^2 - 1\right]\sinh\gamma_m L}{\left[\left(\frac{Z_m}{Z_{0m}}\right)^2 + 1\right]\sinh\gamma_m L + 2\left(\frac{Z_m}{Z_{0m}}\right)\cosh\gamma_m L} \quad (30)$$

$$T_m = \frac{2\left(\frac{Z_m}{Z_{0m}}\right)}{\left[\left(\frac{Z_m}{Z_{0m}}\right)^2 + 1\right]\sinh\gamma_m L + 2\left(\frac{Z_m}{Z_{0m}}\right)\cosh\gamma_m L}. \quad (31)$$

Similarly, when $\partial_x \equiv 0$, one can write the total tangential fields for TE_{0n}-modes inside the waveguide as follows:

$$\vec{E}_n^{\perp} = [E_n^+ \exp(-\gamma_n z) + E_n^- \exp(\gamma_n z)]\sin\left(\frac{n\pi y}{b}\right)\mathbf{e}_x \quad (32)$$

$$\vec{H}_n^{\perp} = \frac{1}{Z_n}[E_n^+ \exp(-\gamma_n z) - E_n^- \exp(\gamma_n z)]\sin\left(\frac{n\pi y}{b}\right)\mathbf{e}_y \quad (33)$$

where

$$\gamma_n = j\sqrt{\omega^2\varepsilon_1\mu_2 - \frac{\mu_2}{\mu_3}\left(\frac{n\pi}{b}\right)^2} \quad (34)$$

$$Z_n = \frac{j\omega\mu_2}{\gamma_n}. \quad (35)$$

The reflection coefficient R_n and the transmission coefficient T_n for the TE_{0n} modes can be defined analogously. The continuity of the tangential fields at $z = 0$ and $z = L$ will then give

$$(1 + R_n)\cosh\gamma_n L - \frac{Z_n}{Z_{0n}}(1 - R_n)\sinh\gamma_n L = T_n \quad (36)$$

$$(1 + R_n)\sinh\gamma_n L - \frac{Z_n}{Z_{0n}}(1 - R_n)\cosh\gamma_n L = -\frac{Z_n}{Z_{0n}}T_n \quad (37)$$

where $\gamma_{0n} = j\sqrt{\omega^2\varepsilon_0\mu_0 - \left(\frac{n\pi}{b}\right)^2}$ and $Z_{0n} = \frac{j\omega\mu_0}{\gamma_{0n}}$. Thus from the above two equations one can obtain similar expressions for the reflection and transmission coefficients for TE_{0n}-modes as (30) and (31) (replace the subscript m with n).

IV. Determination of the Material Parameters

In the inverse problem, it is assumed that one knows the reflection and transmission, and wants to determine the constitutive parameters of the Ω material inside the waveguide.

A. Determination of the Propagation Constant and Impedance

From (28) and (29) one obtains the following equation by eliminating Z_m/Z_{0m}

$$\cosh\gamma_m L = \frac{T_m^2 - R_m^2 + 1}{2T_m} \equiv a_m + jb_m \quad (38)$$

where a_m and b_m are real and measurable.

Let

$$\gamma_m = \alpha_m + j\beta_m \quad (39)$$

where $\beta_m > 0$ (cf. the definition (21)), $\alpha_m \geq 0$ (since we only consider passive media). From (38) one obtains

$$\cosh\alpha_m L\cos\beta_m L = a_m \quad (40)$$

$$\sinh\alpha_m L\sin\beta_m L = b_m. \quad (41)$$

Eliminating the trigonometric functions in the above two equations, one obtains

$$\sinh^4\alpha_m L - (a_m^2 + b_m^2 - 1)\sinh^2\alpha_m L - b_m^2 = 0.$$

Since $\sinh\alpha_m L \geq 0$ (note that $\alpha_m \geq 0$), from the above equation one obtains

$$\sinh\alpha_m L = \sqrt{\frac{1}{2}[a_m^2 + b_m^2 - 1 + \sqrt{(a_m^2 + b_m^2 - 1)^2 + 4b_m^2}]}$$

which gives α_m uniquely

$$\alpha_m = \frac{1}{L} \cdot \sinh^{-1}\sqrt{\frac{1}{2}\left[a_m^2 + b_m^2 - 1 + \sqrt{(a_m^2 + b_m^2 - 1)^2 + 4b_m^2}\right]}. \quad (42)$$

After α_m has been determined, one has (cf. (40) and (41))

$$\cos\beta_m L = \frac{a_m}{\cosh\alpha_m L} \qquad \sin\beta_m L = \frac{b_m}{\sinh\alpha_m L} \quad (43)$$

which gives

$$\beta_m = \tilde{\beta}_m + \frac{2p\pi}{L}, \quad p = 0, 1, 2, \cdots \quad (44)$$

where $\tilde{\beta}_m$, $0 < \tilde{\beta}_m \leq 2\pi/L$, is uniquely determined from (43). However, the integer p should be determined from some other information. If the length L of the Ω sample is sufficiently small, then $p = 0$. Note that the propagation factor $\exp(-\gamma L)$ does not change when the length L changes by a multiple of the wavelength. Much work has been done to eliminate this phase ambiguity in inverse problems by, e.g., making time-domain measurements [12], or making measurements at two or more frequencies [15], or using samples of different sizes [16]. Thus we will not address this aspect in the present paper (in the numerical reconstruction we take the correct value of the integer p from the direct calculation).

After the propagation constant γ_m has been determined from (42) and (43) for the TE_{m0} mode, one can obtain the impedance Z_m for the mode (cf. (28))

$$Z_m = Z_{0m}\frac{(1 + R_m)\cosh\gamma_m L - T_m}{(1 - R_m)\sinh\gamma_m L}. \quad (45)$$

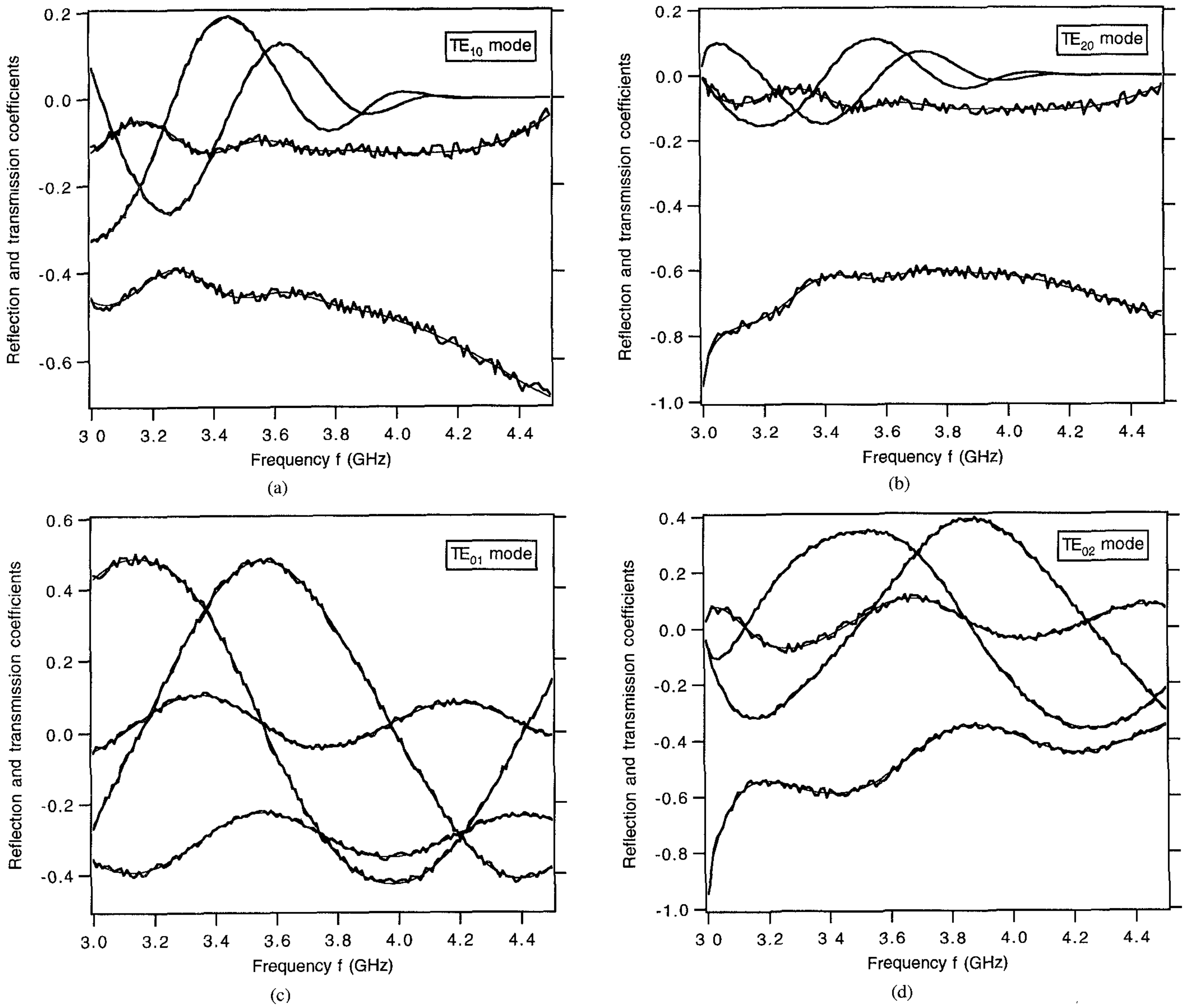

Fig 2. The reflection and transmission coefficients for an Ω sample (with its parameters shown in Fig. 3) in the rectangular waveguide. The size of the Ω sample is $a = b = L = 0.1$ m. The thin solid curves are for the clean data, and the thick solid curves are for the noisy data. (a) TE_{10} mode. (b) TE_{20} mode. (c) TE_{01} mode. (d) TE_{02} mode.

In a completely analogous way, one can determine the propagation constant γ_n and the impedance Z_n for the TE_{0n}-mode.

B. Determination of the Constitutive Parameters

After the propagation constant and the impedance have been determined, one can determine the constitutive parameters of the Ω material as follows:

First we excite the waveguide with the TE_{01} and TE_{0N} $(N \geq 2)$ modes at a frequency ω. From (35) one obtains

$$\mu_2 = \frac{\gamma_n Z_n}{j\omega}, \qquad n = 1. \tag{46}$$

Furthermore, from (34) one has

$$\varepsilon_1 = \frac{\gamma_N^2 - N^2\gamma_1^2}{\mu_2\omega^2(N^2 - 1)} \tag{47}$$

$$\mu_3 = \mu_2\left(\frac{\pi}{b}\right)^2 \frac{1}{\gamma_1^2 + \omega^2\varepsilon_1\mu_2}. \tag{48}$$

Then we excite the waveguide with the TE_{10} and TE_{M0} $(M \geq 2)$ modes at the same frequency ω. From (21) one obtains

$$\mu_1 = \mu_3 \frac{a^2(\gamma_M^2 - \gamma_1^2)}{\pi^2(M^2 - 1)}. \tag{49}$$

Then from (24) one has

$$\Omega = \frac{\gamma_m - j\omega\mu_1/Z_m}{\omega\sqrt{\varepsilon_0\mu_0}}, \qquad m = 1. \tag{50}$$

Finally, from (21) one obtains

$$\varepsilon_2 = \frac{\varepsilon_0\mu_0\Omega^2}{\mu_1} + \frac{\frac{\mu_1}{\mu_3}\left(\frac{m\pi}{a}\right)^2 - \gamma_m^2}{\omega^2\mu_1}, \qquad m = 1. \tag{51}$$

Since ε_3 does not affect the reflection or transmission for any TE_{m0}- or TE_{0n}-mode, one can not determine ε_3 by only

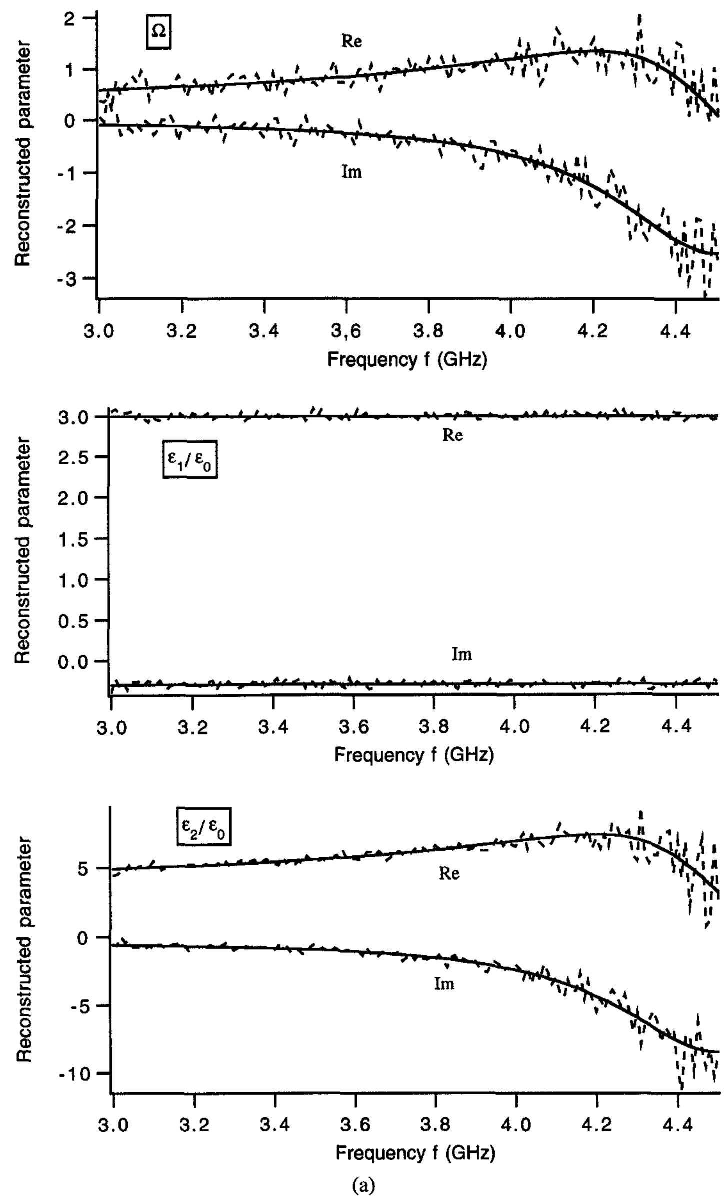

Fig. 3. The reconstruction of the dispersive parameters using the noisy reflection and transmission data shown in Fig. 2.

measuring the reflection and transmission for TE_{m0} and TE_{0n}-modes. One may hope to determine ε_3 by repositioning the Ω sample in the waveguide, and measuring the corresponding reflection and/or transmission coefficients for certain decoupled mode. However, one can show that if the Ω sample is repositioned in the waveguide with the third principal axis of the bi-anisotropic Ω sample in the $x(y)$ direction, then the only possible decoupled modes are TE_{m0} (TE_{0n})-modes. Furthermore, from the analysis of these modes one can easily see that the reflection or transmission for any possible decoupled mode in any case does not depend on ε_3. Thus the remaining parameter ε_3 should be determined from some other measurements, e.g., free-space measurement of the reflection for a normally incident plane wave (propagating in x direction) with electric field polarized in z direction. The corresponding reflection coefficient is [9]

$$r = [1 - \exp\left(j4\pi a/\sqrt{\varepsilon_3\mu_2}\right)]\frac{\tilde{r}}{1-\tilde{r}^2}$$

where a is the thickness of the sample and

$$\tilde{r} = \frac{\sqrt{\varepsilon_0/\mu_0} - \sqrt{\varepsilon_3/\mu_2}}{\sqrt{\varepsilon_0/\mu_0} + \sqrt{\varepsilon_3/\mu_2}}.$$

Note that in principle it may be possible to determine all the constitutive parameters by free-space measurement of the reflection and transmission for plane waves obliquely incident on

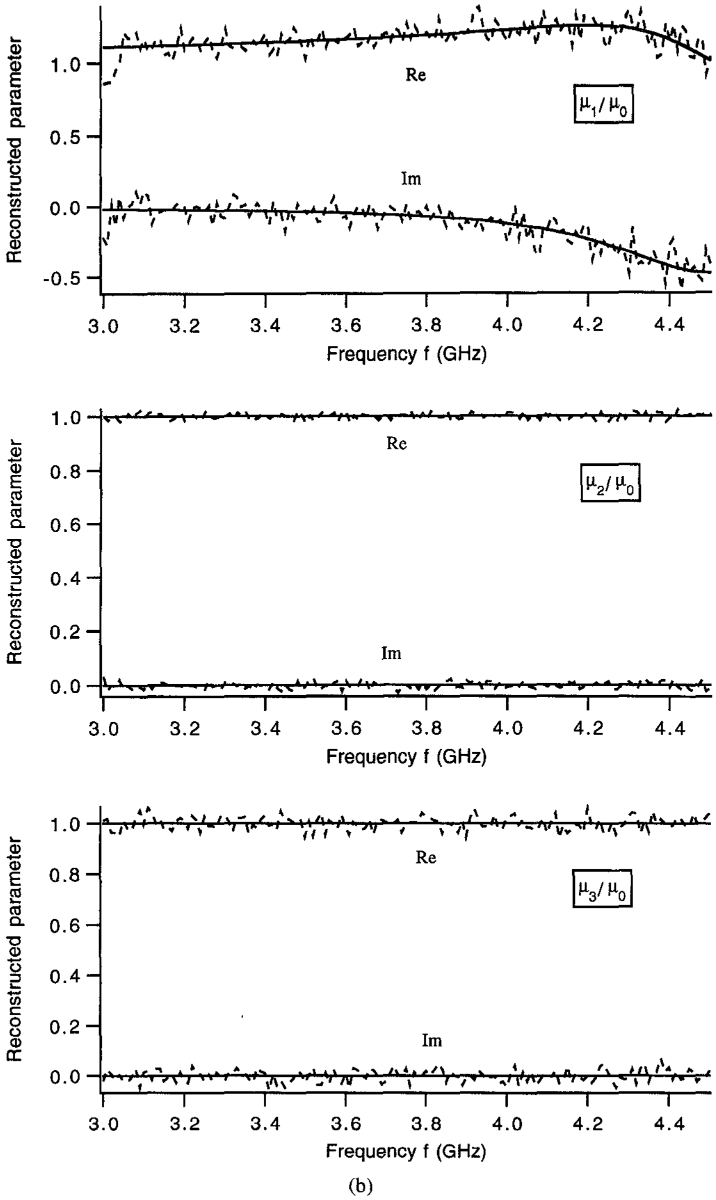

Fig. 3. *(Continued)*

an Ω slab with several different incident angles. However, it is difficult to measure the reflection and transmission accurately in free space due to secondary diffraction effects. Furthermore, the expressions for the reflection and transmission coefficients for an Ω slab are very complicated for the oblique incidence case [9], and are not attractive for use to determine the constitutive parameters explicitly. Thus reconstruction from waveguide measurements should be used when possible.

V. Numerical Results

Based on the results reported in [17] and [18], we choose the frequency dependence of the parameter Ω as shown in Fig. 3. ε_2 and μ_1 will consequently become frequency-dependent and larger in amplitudes (compared to the other diagonal elements of the permittivity or permeability tensor) due to the cross-coupling between the electric and magnetic fields, if the host material is isotropic and frequency-independent (in the frequency range we consider here) before the insertion of the Ω-shaped metal loops. Thus we choose the constitutive parameters of the Ω sample as shown in Fig. 3 for the numerical test. The size of the Ω sample is $a = b = L = 0.1$ m. In the vacuum region $z < 0$, the cutoff frequency is $f_c = m/(2a\sqrt{\varepsilon_0\mu_0})$ (or $n/(2b\sqrt{\varepsilon_0\mu_0})$) for TE_{m0} (or TE_{0n} mode). We choose the frequency range between the cutoff frequencies for the second and third modes, so that only the first and second modes can be excited in this frequency range (in this case the frequency range is $3 \sim 4.5$ GHz). The corresponding reflection and transmission coefficients for TE_{m0} $(m = 1, 2)$ and TE_{0n} $(n = 1, 2)$ modes are calculated using (30) and (31), and are shown by the thin solid curves in Fig. 2 (note that these reflection and transmission coefficients are independent of the parameter ε_3). To test the stability of the reconstruction scheme, we have added 3% (with respect to the absolute value of the reflection or transmission coefficient) of random noises to both the real and imaginary parts of the reflection and transmission coefficients. The noisy data are shown by the thick solid curves in Fig. 2. Using the reconstruction scheme described in Section IV, the reconstructed parameters and the true parameters essentially coincide on the scale of Fig. 3 if the clean data of the reflection and transmission are used. The reconstruction of the constitutive parameters using the noisy reflection and transmission data is shown by the dashed curves in Fig. 3. Since we choose the dispersive parameters to be much lossier in the frequency range $4.0 \sim 4.5$ GHz, the transmission coefficients become very small in the range $4.0 \sim 4.5$ GHz (see Fig. 2(a) and (b)). As a consequence, the reconstruction of the parameters (shown by the dashed curves in Fig. 3) becomes more sensitive to the noise in the range $4.0 \sim 4.5$ GHz (this is because the determination of the propagation constant becomes more sensitive to the random noise if the transmission coefficients become very small, cf. (38)).

VI. Conclusion

In the present paper all the dispersive parameters except ε_3 of a bi-anisotropic Ω material have been reconstructed using the reflection and transmission data in a rectangular waveguide for some TE_{m0} and TE_{0n}-modes (with two different values of m and two different values of n at each frequency). The analytical expressions for the reflection and transmission coefficients for these modes have been given. Numerical results for the reconstruction have been presented, and the reconstruction scheme has been tested with noisy data.

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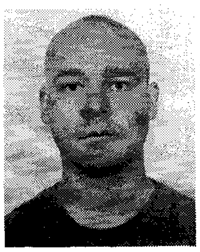

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